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Optimal Overhaul-Replacement Policies For a Repairable Machine Sold with Warranty

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Abstract. This research deals with an overhaul-replacement policy for a repairable machine sold with a free replacement warranty (FRW), which will be used for a finite horizon $\mathcal{T}$ and evaluated at fixed interval s. At each evaluation point, the buyer considers three alternative decisions, i.e. (1) keep the machine, (2) overhaul it, or (3) replace it with a new identical one. If the machine fails during the warranty period, this is rectified at no cost to the buyer. Any failure occurring before and after the expiry of the warranty is restored by minimal repair. An overhaul-replacement policy was formulated using a dynamic programming approach. The results show that overhaul rejuvenation may extend the machine life cycle and delay the replacement decision. In contrast, the warranty stimulates early machine replacement and in so doing increases the replacement frequency for a certain range of replacement costs. This implies that in order to minimize the total ownership costs over $\mathcal{T}$, the buyer needs to consider the minimal repair cost reduction due to the rejuvenation effect of an overhaul as well as the warranty benefit due to replacement. Numerical examples are presented for both illustrating the optimal policy and describing the behavior of the optimal solution.

Keywords: *dynamic programming; minimal repair; overhaul; replacement; warranty.*

1 Introduction

We consider a machine that is used as a means of production. Failure of the machine will cause losses, either because of delayed completion, decreased production, or process inefficiency [1-3]. In the case where the machine has an increasing failure rate, it is reasonable to overhaul or replace it before its performance falls below the standard. Optimal replacement models can be classified in many different categories [4-6].

A warranty is a contractual agreement between a manufacturer and a consumer to establish liability in case of a premature failure of an item or inability to perform its intended function. One type of warranty that is usually offered for

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repairable products is the free replacement warranty (FRW) [7]. If the machine fails within the FRW warranty period, the manufacturer agrees to repair or replace the failed product at no cost, such that the buyer only suffers the cost due to interruption of production. Most research on warranties focuses on reducing the warranty cost, which is an issue of great interest to manufacturers [8]. They deal mainly with the optimal strategy from the manufacturer's perspective, i.e. to minimize the expected warranty cost over the warranty period [9-11]. Meanwhile, [12] and [13] have shown that the warranty has a significant impact on the total operation and maintenance costs borne by the buyer. The optimal strategy for the buyer should be determined under a life-cycle context [14]. There are researches that address this issue from the viewpoint of the consumer. To mention a few among them are the following. Optimal post-warranty preventive maintenance policies have been developed to minimize the expected long-run maintenance cost per time unit [15]. The customer who owns the equipment may offer improved preventive maintenance to negotiate a better warranty contract [16].

Most researches on maintenance and replacement are analyzed under an infinite horizon assumption. However, most system life cycles are finite, in which case it is important to consider optimal policies under a finite horizon. Three common infinite horizon models, i.e. periodic replacement with minimal repair, block replacement and simple replacement, have been modified to finite-span replacement models in [17]. A threshold on the current system state and a threshold on the residual life cycle have been considered in modeling finite horizon replacement decisions for a multi-state system in [18].

An equipment replacement model for a system that is required only for a specified length of time in order to fulfill a specific contract has been developed in [19]. The work was motivated by an interest in studying the validity of applying an infinite-horizon solution–i.e. to replace an asset at its economic life–to a finite-horizon problem. Formulated as an integer-knapsack model and solved by using dynamic programming, it presents a different approach to bound the number of times an asset is utilized at its economic life during the considered period. The study was intended to examine the application of asset economic life in a finite-horizon equipment replacement problem by considering the time value of money. The authors developed a simple bounding technique in dynamic programming. None of the papers cited above considers the overhaul option and the warranty benefit in a replacement decision.

Our research deals with optimal overhaul-replacement policies for a machine sold under warranty from the buyer's point of view. Our work is motivated by an interest in the interaction effects of warranty advantage and overhaul benefit toward the buyer's optimal policy. In a previous research, we have modeled an optimal replacement model for a repairable machine sold with warranty under a finite horizon planning [20]. In the present study, we extended this by adding the overhaul consideration into the buyer's decision. Applying a periodic decision approach, we further developed a dynamic programming model that has been used in many equipment replacement optimization studies ([21,22]). The model is used to minimize the total expected ownership costs over T, which consists of expected minimal repair cost, overhaul cost and replacement cost.

This paper is organized as follows. Section 1 describes the background of the research and indicates the research gap. In Section 2 we present system characterization. In Section 3 we present model formulation, along with the analysis to prove the existence of an optimal solution and discuss some underlying situations for the overhaul and replacement decision. In Section 4 we present some numerical examples. Finally, in Section 5 we provide conclusions and discuss the extension of our work.

2 System Characterization

We consider a repairable machine sold under a free replacement warranty (FRW). The machine is planned to operate as a means of production for a finite horizon, $\mathcal{T}$ ($\mathcal{T}<\infty$). To maintain the machine's performance, it is evaluated N times during $\mathcal{T}$ (Figure1). The interval between evaluation points s ($s<\mathcal{T}$) is constant, so $\mathcal{T}= N.s$ where N takes integer values.

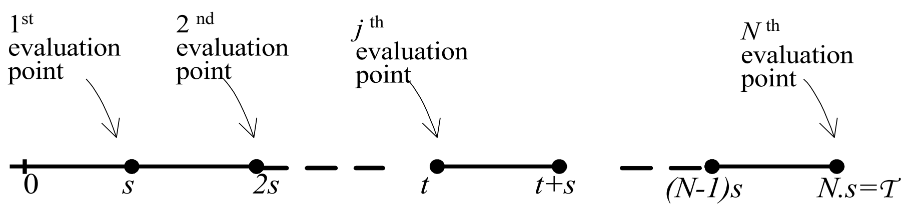

Figure 1 Evaluation points at each end of operation period *s*.

At j=0 a new machine begins to operate. At each evaluation point j (j=1,…N-1) the consumer has the alternatives to (1) keep the machine, (2) overhaul it, or (3) replace it with a new identical one. We assume that machine operation will terminate at the N^{th} evaluation point.

2.1 Failure Modeling

Machine failures are modeled using a black-box approach, so the damage mechanisms are not specifically considered [23]. The occurrence of failures along the time axis is represented by a point process with intensity function $\lambda(\tau)$, an increasing function of τ. Any failure is detected instantly and minimally repaired. Under minimal repair a failed machine is restored to its condition immediately before the failure. Thus the conditional failure intensity function is unaffected by each failure. This is appropriate for the situation of a multi-component machine [24]. The rectification of the failed components has a negligible effect on the remaining components. If the time required to conduct minimal repair is negligible, then the machine continuously experiences deterioration and the occurrence of failures during s follows a non-homogeneous Poisson process [25]. Let t represent the age of the machine. Assuming minimal repair occurrence, the expected number of failures during s, $h(t,t+s)$, is given by (1):

$$h(t,t+s)=\int_t^{t+s}\lambda(\tau)d\tau \tag{1}$$

2.2 Warranty

The machine is sold under a free replacement warranty with a warranty period of w. w is an integer multiplication of s (w=$n.s$). Any failure that occurs during w will be repaired by the seller with no charges to the buyer. In such situation the buyer still bears some cost, c_1, due to less desirable quality products produced during the transient condition after the machine was repaired minimally. After the warranty expires the consumer will bear the total repair costs as well as losses due to disruptions in the production process, c_2, whereby, $c_1<c_2$. If γ denotes the minimal repair cost per failure incurred, then the value of γ depends on the machine age t as follows:

$$\gamma=\begin{cases} c_1, t<w \\ c_2, t\geq w \end{cases} \tag{2}$$

2.3 Overhaul Modeling

Overhaul is defined as a planned systematic effort (check, detect, treat or replace components) in order to maintain machine performance at a certain level. A number of models assume overhaul to improve the performance of the machine by rejuvenation. Thus, after overhaul the machine has a virtual age that is younger than the actual age, or a lower failure intensity but not to the point that the equipment is as good as new [26]. Two virtual-age models have been examined in [27]. One of these is a permanent rejuvenation model, which assumes that due to overhaul the difference between actual age and virtual age is always the same. Using this approach, in this research we consider overhaul as an action that decreases the machine's age with a constant reduction δ.

Overhaul will not restore the machine to be as good as a new machine that is still under warranty. Therefore overhaul may only be carried out after the machine age t, $t>w+\delta$. We express δ as an integer multiplication of operation interval s ($\delta = k.s$, k = 1,2,...). Furthermore, we assume that the time required to conduct overhaul is considered to be negligible. Consequently, any overhaul at j for a t-old machine will immediately reduce the machine's age to t-δ. The overhaul cost is c_3, which is higher than the minimal repair cost. As a result $c_1<c_2<c_3$.

2.4 Decision Alternatives

At any j, there are three decision alternatives to choose from. If we choose to keep operating the machine then, due to aging, machine failure as well as minimal repair costs will increase. If the decision is to overhaul then the cost for the machine overhaul will be incurred but it also reduces the failure intensity due to machine rejuvenation. Lastly, if the decision is to replace the machine then a notable cost incurs for replacing the machine, however, it is restored to a new one and this significantly reduces repair costs during the warranty period. Assuming the machine's operating cost is constant, the expected total ownership costs are the accumulation of expected minimal repair costs, overhaul costs and replacement costs over $\mathcal{T}$.

The mathematical model is formulated to minimize the expected total costs of machine ownership throughout $\mathcal{T}$. A constraints that was considered is the requirement to be able to operate the machine during the whole considered planning horizon. Minimizing the total machine ownership costs is done by choosing a decision at every j (j=1,…N), which can be expressed as a function of the sequential decisions taken during the planning period, represented by the following steps.

3 Dynamic Programming Formulation and Analysis

The model is developed from the buyer's point of view and system performance is measured by the total ownership costs over $\mathcal{T}$. We apply dynamic programming to formulate the overhaul-replacement problem of a warranted machine [28]. Let x_j denote the action taken at j, j=1,2…N-1. The stages in this problem refer to evaluation points j. Each stage has a number of states that refer to machine age t. We consider two types of costs, i.e. minimal repair cost (within warranty period c_1 and after warranty expiry c_2), decision cost i.e. overhaul cost c_3, and replacement cost c_4. We assume that the highest cost is replacement cost c_4, therefore $c_1<c_2<c_3<c_4$. Overhaul is modeled as an action that generates machine rejuvenation with constant age reduction. If the machine is overhauled at age t then the machine's virtual age after overhaul will be t-δ,

where δ is constant [29]. Figure 2 shows the relationship between state t at j, decision x_j and its contribution to the machine's state at j+1.

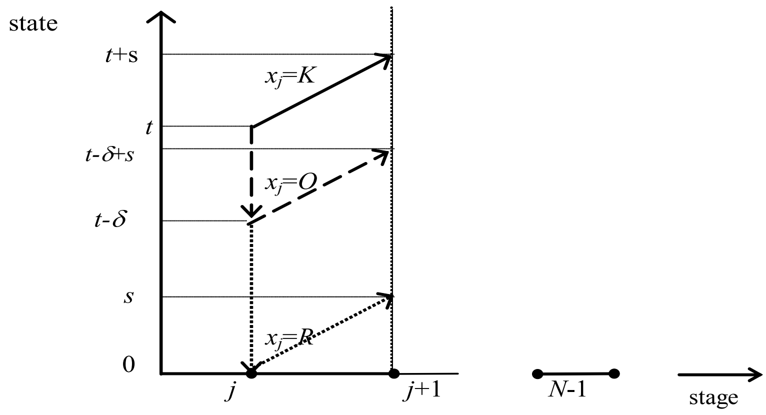

Figure 2 Relationship between machine state t at stage j, decision x_j and its contribution to the j+1th state.

If at j we have a machine with an age of t and the decision is to keep it (K) then at the next evaluation point (j+1), the machine age will be $t+s$. If the decision is to overhaul (O) at j then we start operation at j with machine age t-δ, where δ is the age reduction due to rejuvenation. Therefore at the next evaluation point (j+1), the machine age will be t-δ+s. Lastly, if the decision is to replace (R) then at j the machine age returns to zero and at the next evaluation point (j+1) the machine age will be s. At time j=0, the buyer starts with a new machine and will determine the sequential decision at j=1,2,N-1, i.e. either to keep operating, to overhaul, or to replace the machine. In general, the buyer will want to maximize the benefit of the warranty that is attached to the replacement decision. Therefore during the warranty period, the buyer's decision will always be to keep the machine, while replacement of the machine should be carried out in such a way that at j=N the warranty of the machine on hand has expired. If the machine age is t, then the decision at j, x_j, is given by:

$$x_j = \begin{cases} Keep \quad (K) & ,\forall t \qquad\quad ,j = 0,...,N-1 \\ Overhaul\,(O) & ,t \ge w+\delta\,, j = w+\delta,...,N-w-\delta \\ \mathrm{Re}\,place\,(R) & ,t \ge w \qquad\; ,j = w,...,N-w \end{cases} \tag{3}$$

Next we obtain the dynamic programming formulation for the overhaul-replacement problem. Let $F_j(t)$ be the total cost of running a machine with age t from j to N. Then $F_j(t)$ is the summation of keep, overhaul, or replacement costs at j, the associated minimal repair cost, and the lowest total ownership costs for the remaining stages (stage j+1 onward). Moreover, we define $F_j^*(t)$ as the best value of $F_j(t)$ for the optimal decision, x_j^*. For $e(t)$ is the machine salvage value with an age t by using (1), (2), and (3), the associated costs at j for each x_j are presented in Table 1.

Table 1 Decisions at j, x_j and the associated costs for the current stage and onward.

x_j	**Decision Cost and the Associated Minimal Repair Costs at *j***	**State at *j*+1**	**Total Costs at *j* and the Remaining Stages**
K	$\gamma h(t,t+s)$	$t+s$	$h(t,t+s)+F_{j+1}^*(t+s)$
O	$c_3+\gamma h(t-\delta,t-\delta+s)$	$t-\delta+s$	$c_3+\gamma h(t-\delta,t-\delta+s)+F_{j+1}^*(t-\delta+s)$
R	$c_4-e(t)+\gamma h(0,s)$	s	$c_4-e(t)+\gamma h(0,s)+F_{j+1}^*(s)$

Our purpose is to seek the optimal sequential decisions (keep, overhaul, or replace) that minimize $F_0^*(t)$. From Table 1 we define $F_j^*(t)$ as a recursive equation at stage j for a machine with age t that minimizes the total ownership costs at stage j and onward by choosing x_j. The dynamic programming formulation of optimal overhaul-replacement policies is then given by:

$$F_j^*(t)=\min\begin{cases} x_j=K & \gamma h(t,t+s)+F_{j+1}^*(t+s) \\ & ,\forall t, j=0,...,N-1 \\ \\ x_j=O & c_3+\gamma h(t-\delta,t-\delta+s)+F_{j+1}^*(t-\delta+s) \\ & ,t\geq w+\delta, j\leq N-w-\delta \\ \\ x_j=R & c_4-e(t)+\gamma h(0,s)+F_{j+1}^*(s) \\ & ,t\geq w, j\leq N-w \end{cases} \tag{4}$$

At the end of the planning period (j=N), $F_N^*(t)$, the boundary condition, is equal to the salvage value of the machine, given by:

$$F_N^*=-e(t) \tag{5}$$

A backward approach is used to obtain the optimal ownership costs at j and its optimal decision x_j^* from j=N-1, j=N-2, up to j=1. Once x_1^* is obtained the optimal policy can be obtained. The optimal policy in stage one (j=1) is used to find the state at j=2. Then we can find the optimal policy in stage two, x_2^*, which is used to find the state at j=3. This approach is done up to j=N. Finally, we obtain the optimal sequential decisions from j=1 up to j=N that give the minimum total ownership costs over $\mathcal{T}$, $F_0^*(t)$.

This simple recursive pattern of dynamic programming results in a relatively short planning horizon problem along with a slight range of model parameters that can be solved in Microsoft Excel. For solving a problem with a longer planning horizon and considerable variation in model parameters, we need to build a computer program to obtain the optimal policy. In the following section, we carry out a model analysis to prove the existence of an optimal solution.

3.1 Existence of Optimal Solution

For F_j^*, given by (4), there exists an optimal solution that minimizes $F_0^*(t)$ given by the decision policy $\pi^* = \{x_0^*, x_1^*, ..., x_{N-1}^*\}$. We use an induction proof to show the existence of an optimal solution for our model. First we specify the space state at any stage, i.e. the machine's age at any j (j=1,2,…N-1) as t_j, $t_j \in T_j$. $T_j = \{s, 2s, \dots js\}$.

We denote $G_j(t_j, x_j)$ as the current stage's costs, i.e. the decision cost at j and the associated expected minimal repair cost:

$$G_j(t_j,x_j) = \begin{cases} \gamma h(t_j, t_j + s) & ,x_j = K \\ c_3 + \gamma h(t_j - \delta, t_j - \delta + s) & ,x_j = O \\ c_4 - e(t_j) + \gamma h(0,s) & ,x_j = R \end{cases} \tag{6}$$

The state in the successive stage t_{j+1} depends on the decision taken at the current stage. We denote:

$$t_{j+1} = \begin{cases} t_j + s & ,x_j = K & ,\forall t_j & ,j = 0,1,...,N-1 \\ t_j - \delta + s & ,x_j = O & ,t_j \geq w + \delta & ,j = w + \delta,...,N - w - \delta \\ s & ,x_j = R & ,t_j \geq w & ,j = w,..,N \end{cases} \tag{7}$$

Since w and δ are defined as integer multiplications of s, all feasible x_j may facilitate every movement from any t_j to one t_{j+1}.

Using (6) and (7) we rewrite (4), the best total ownership costs at a particular j, as:

$$F_j^*(t) = F_j(t_j, x_j^*) = \min_{x_j \in \{K,O,R\}} (G_j(t_j, x_j) + F_{j+1}^*(t_{j+1})) \tag{8}$$

Using the principle of optimality of dynamic programming we can show that for any j, (j=0,1, 2,…N-1) if there exists an x_j^* for a certain t_j that satisfies (8), then we can find x_{j-1}^*.

For *j*=*N*;

As N is the last stage, there is no cost to be considered and the machine is sold. To be specific, at the end of the planning horizon, the warranty should be expired, so possible states t_N are finite, i.e. $t_N \in T_N, T_N = \{w, w+s, ..., Ns\}$.

As the salvage value of the machine depends on its age, we can rewrite (5) as:

$$F_N^* = F_N^*(t_N) = -e(t_N), t_N \in T_N \tag{9}$$

We proceed to show the existence of an optimal solution at j=N-1 by using $F_N^*(t_N)$.

For *j*=*N*-1;

At j=N-1, the state spaces t_{N-1} are finite, i.e. $t_{N-1} \in T_{N-1}, T_{N-1} = \{w-s, w, ..., (N-1)s\}$. Using (8), the optimal expected cost at stage N-1 can be expressed as:

$$F_{N-1}(t_{N-1,} x_{N-1}^*) = \min_{x_{N-1} \in \{K,O,R\}} (G_{N-1}(t_{N-1}, x_{N-1}) + F_N^*(t_N)) \tag{10}$$

From any $t_{N-1} \in T_{N-1}$ there exists at least one feasible $x_{N-1} \in \{K, O, R\}$ that facilitates movement to one $t_N \in T_N$ in the subsequent stage. Hence it follows that from all feasible solutions x_{N-1} there is at least one x_{N-1}^* that gives the minimum ownership costs for the last period to go. As a result, at j=N-1 the optimal solution x_{N-1}^* can be obtained for all $t_{N-1} \in T_{N-1}$.

Continuing the backward process until stage 0, we will certainly obtain optimal decision sequences for the remaining stages, i.e. $x_{N-2}^*, x_{N-3}^*, ..., x_0^*$. Finally, we have $\pi^* = \{x_0^*, x_1^*, ..., x_{N-1}^*\}$ that minimizes $F_0^*(t)$.

3.2 Necessary Condition for Overhaul

For any j and t ($t \geq w + \delta$) we perform overhaul with cost c_3, which will reduce the age of the machine from t to t-δ. We consider a non-warranted situation

(w=0 and c_1=c_2). Using Eq. (4), we obtain the necessary condition at j where overhaul is better than keep operating (11).

$$(\gamma h(t,t+s)-\gamma h(t-\delta,t-\delta+s))+(F^*_{j+1}(t+s)-F^*_{j+1}(t-\delta+s))>c_3 \quad (11)$$

As $h(t)$ and $F_j(t)$ are increasing functions in t [20], the left hand side of (11) also increases in t. The first term on the left hand side of (11) shows minimal repair cost reduction due to overhaul only at j, while the second term represents the additional benefit in the remaining stages. Note that if $\delta \to 0$, then doing overhaul or keep operating the machine at j are equally attractive. To accomplish (11), the rejuvenation effect δ has to be sufficiently large to produce minimal repair cost reduction both at j and the remaining stages. Using Eq. (4), we also obtain the necessary condition at j where overhaul is better than replacement (12).

$$(c_4-e(t))-(\gamma h(t-\delta,t-\delta+s)-\gamma h(0,s))-(F^*_{j+1}(t-\delta+s)-F^*_{j+1}(s))>c_3 \quad (12)$$

For δ approaches t, the machine's condition after overhaul is about as good as new and the second and the third term on the left hand side of (12) nearly turn to zero. In this case, the only factor that prevents accomplishment of (12) is a high salvage value, $e(t)$, of the machine on hand. For a non-warranted situation, the decision to overhaul at j is only attractive if the rejuvenation effect is significant relative to c_3. One factor that may have the opposite effect is the second-hand resale value of the machine on hand.

3.3 Interaction Effect of Overhaul and Warranty

The benefit of the warranty stimulates early replacement [20]. We develop the necessary condition at j for replacement is better than overhaul using (4) and (2). For w=1 we obtain (13).

$$(c_2 h(t-\delta,t-\delta+s)-c_1 h(0,s))+(F^*_{j+1}(t-\delta+s)-F^*_{j+1}(s))>(c_4-e(t))-c_3 \quad (13)$$

The first term and the second term on the left hand side of (13) is always non-negative, since $h(t)$ and $F_j(t)$ are increasing functions in t, and $c_2>c_1$. Consequently, the necessary condition for choosing replacement at j can be accomplished if one of the following conditions is satisfied.

$$c_4-e(t)-c_3<c_2 h(t-\delta,t-\delta+s)-c_1 h(0,s) \quad (14)$$

$$c_4-e(t)-c_3<(F^*_{j+1}(t-\delta+s)-F^*_{j+1}(s)) \quad (15)$$

The distinctive parameters of the model developed are the overhaul rejuvenation effect δ and the warranty benefits represented by w and c_1/c_2. Therefore, to discuss the underlying buyer's decision to replace we consider only Eq. (14).

The value of the right hand side of (14) is associated with the disparities of c_1 and c_2, which indicate how significant minimal repair cost is to the buyer compared to the losses due to process disruption. Reduction of the left hand side of (14) may happen due to a decrease of c_4 or an increase of $e(t)$ as well as an increase of c_3. This situation shows the buyer's underlying decision in choosing replacement, i.e. a reasonable price for a new machine along with an expensive minimal repair cost or a good salvage value of the machine. In addition, if the overhaul results are insignificant and costly, then the buyer's best decision at j is to choose replacement.

4 Numerical Examples

In this section, we present some numerical examples to illustrate the optimal solutions of the model developed. We consider that the machine has an increasing failure rate, which can be represented by a power law function, $\lambda(\tau)$:

$$\lambda(\tau) = \alpha\beta \cdot \tau^{\beta-1},\ \tau>0,\ \alpha>0,\ \beta>1 \tag{16}$$

The machine will be used for the next twelve years, has a two-year warranty and will be evaluated each year. Hence, we have $\mathcal{T}=12$, $N=12$, $s=1$, and $w=2$. The failure intensity function parameter is $\alpha=2$. The replacement cost c_4 is 1250. The machine's price decreases by 40% within the first year and in each subsequent year the price will fall by 15%. Then, the resale price of the machine with age t is:

$$e(t) = 40\% \times c_4(1 - 15\%)^{t-1}, t = 1,2,\ldots N \tag{17}$$

Overhaul cost c_3 is 250 and machine rejuvenation δ due to overhaul is two years, $\delta = 2$. The minimal repair cost per failure occurrence during warranty c_1 is 171. After the warranty has expired minimal repair cost c_2 is 190.

4.1 Effect of Variations in β

To show how the model solutions respond to the different failure rates we use several values of β (β = 1.20, 1.25, 1.35). The optimal policies are shown in Table 2 and Figure 3. The optimal solutions obtained from different values of β show that the decision to overhaul is only considered for a machine with a low failure intensity. For a machine with a higher failure rate it is cheaper to perform frequent replacement than to do overhaul and keeping the machine on hand.

4.2 Effect of Variations in δ

To demonstrate how rejuvenation effect δ influences the optimal policies, we apply different values of δ (δ =1, 2, 3) by using β = 1.25 but keep the same value of the other parameters. The results are shown in Table 3 and Figure 4. An increasing rejuvenation effect of overhaul leads to an increasing overhaul frequency over $\mathcal{T}$ and this results in significantly lower total ownership costs.

Table 2 Optimal policies x_j for β = (1.20, 1.25, 1.35).

β	Optimal policies at j, x_j^*											Optimal cost
	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*	
1.20	*K*	*K*	*K*	***O***	*K*	***O***	*K*	***O***	*K*	*K*	*K*	7101.86
1.25	*K*	*K*	*K*	*K*	***R***	*K*	*K*	*K*	***O***	*K*	*K*	7725.58
1.35	*K*	*K*	***R***	*K*	*K*	***R***	*K*	*K*	***R***	*K*	*K*	8615.43

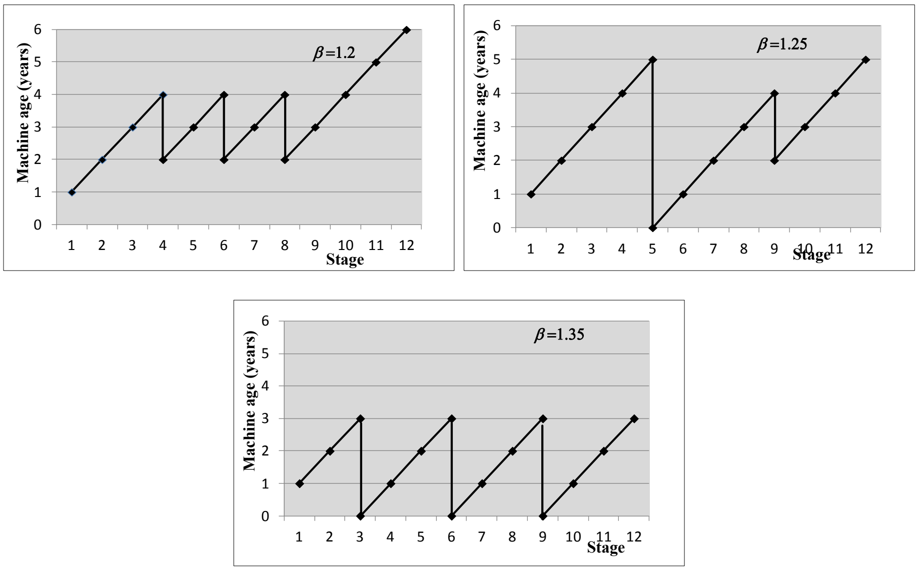

Figure 3 Machine age at stage j due to decision x_j for several values of β.

Table 3 Optimal policies x_j for δ = (1, 2, 3).

δ	**Optimal policies at j, x_j^***											**Optimal cost**
	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*	
1	*K*	*K*	***R***	*K*	*K*	*K*	*K*	***R***	*K*	*K*	*K*	7756.64
2	*K*	*K*	*K*	*K*	***R***	*K*	*K*	*K*	***O***	*K*	*K*	7725.58
3	*K*	*K*	*K*	*K*	***O***	*K*	*K*	***O***	*K*	*K*	***O***	7611.09

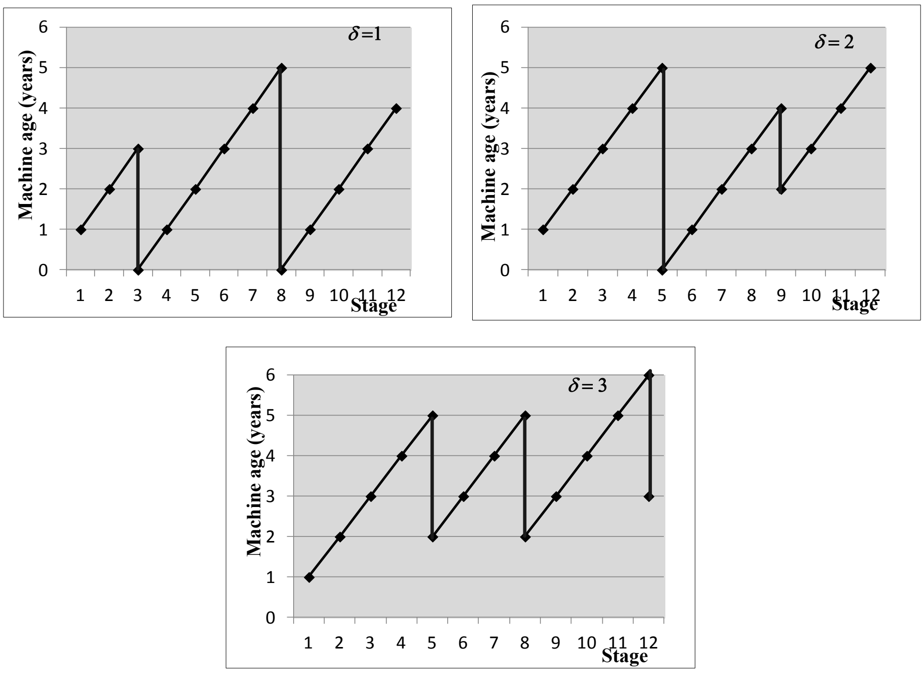

Figure 4 Machine's age at stage j due to decision x_j for several values of δ.

4.3 Effect of Variations in *w*

We represent the benefit of the warranty using two parameters. Firstly, the warranty duration w, and secondly, the ratio of minimal repair costs during and after the warranty period, $r = c_1/c_2, r < 1$. To show how the model solutions respond to different lengths of the warranty duration (w=1,2,3), we use $\beta = 1.25$, while other parameters are kept the same. Table 4 and Figure 5 show the optimal solutions.

Table 4 Optimal policies x_j for w=(1,2,3).

W	**Optimal decisions at *j*, x_j^***											**Optimal cost**
	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*	
1	*K*	*K*	***O***	*K*	***O***	*K*	***O***	*K*	***O***	*K*	*K*	7286.32
2	*K*	*K*	*K*	*K*	***R***	*K*	*K*	*K*	***O***	*K*	*K*	7725.58
3	*K*	*K*	*K*	***R***	*K*	*K*	*K*	***R***	*K*	*K*	*K*	7577.66

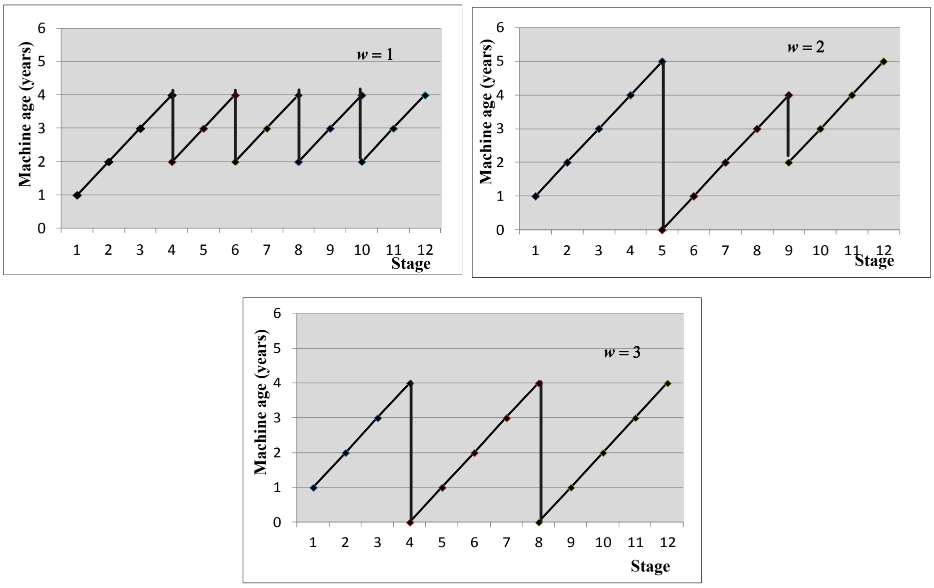

Figure 5 Machine's age at stage j due to decision x_j for several values of w

The longer the warranty period, the higher the benefit obtained by the buyer and hence it will stimulate the buyer to buy a new machine and induce early replacement. Using the model parameters above, if the seller only offers a one-year warranty then the buyer's optimal policy is to overhaul the machine four times with no replacement during twelve years. But if the seller increases the warranty period to three years, then the buyer's optimal policy is to replace the machine twice during the planning horizon considered (in the fourth and eighth year). From Figure 4 and Figure 5 we can show that rejuvenation due to machine overhaul and warranty length have the opposite effect on the buyer's decision at j.

4.4 Effect of Variations in *r*

We now want to show how the model solutions respond to different values of r $(r = c_1/c_2)$. Table 5 and Figure 6 show optimal policies for β = 1.20 and r=(65%, 70%, 90%). During the warranty period, the minimal repair cost is fully borne by the seller, while the buyer only experiences undesirable product quality cost c_1 caused by production process disturbances. Figure 4 describes how different losses due to process disruption influence the optimal solution. The greater the disparities between c_1 and c_2 or the smaller r indicates that the minimal repair cost is more considerable compared to the loss due to production process disruption. As a result, free minimal repair offered by the seller during the warranty period is quite valuable from the buyer's point of view. In such a situation, for a certain range of replacement cost c_4, the buyer's optimal solution is to increase the replacement frequency over $\mathcal{T}$.

Table 5 Optimal policies x_j for r = (65%,70%,90%).

r	Optimal decisions at j, x_j^*											Optimal cost
	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*	
65%	*K*	*K*	***R***	*K*	*K*	*K*	*K*	***R***	*K*	*K*	*K*	6679.60
70%	*K*	*K*	*K*	*K*	*K*	***R***	*K*	*K*	***O***	*K*	*K*	6807.82
90%	*K*	*K*	*K*	***O***	*K*	***O***	*K*	***O***	*K*	*K*	*K*	7101.86

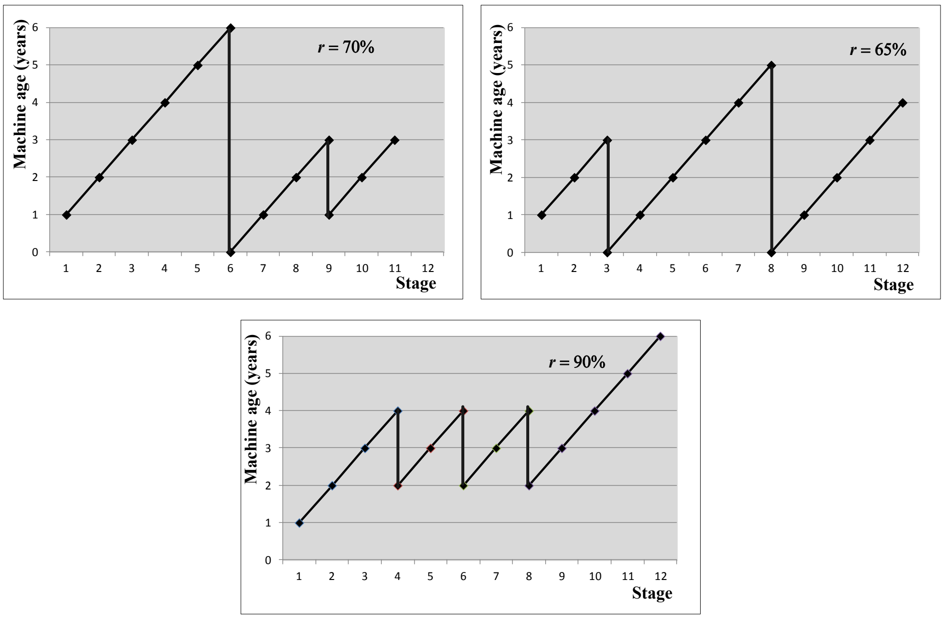

Figure 6 Machine's age at stage j due to decision x_j for several values of r.

4.5 Effect of Variations in $\mathcal{T}$

The final numerical examples show how the model solutions respond to different lengths of $\mathcal{T}$. Table 6 shows optimal policies for β = 1.25, r=90%, and $\mathcal{T}$=(8,10,11,12). A sequential solution for a finite-horizon problem is always influenced by the closeness to the beginning of the period considered. Moreover, the model developed also limits the decisions near the end of the planning period. Figure 7 shows that the optimal decision obtained is sensitive to changes in the planning period considered.

Table 6 Optimal policies x_j for $\mathcal{T}$=(8,10,11,12).

$\mathcal{T}$	x_1^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*	x_{12}^*	**Optimal cost**
8	...		*O*	*K*	*O*	*K*	--					4743.87
10	...		*K*	*R*	*K*	*K*	*K*	*K*	--			6229.75
11	...		*O*	*K*	*O*	*K*	*O*	*K*	*K*	--		6977.36
12	...		*K*	*R*	*K*	*K*	*K*	*O*	*K*	*K*	--	7725.58

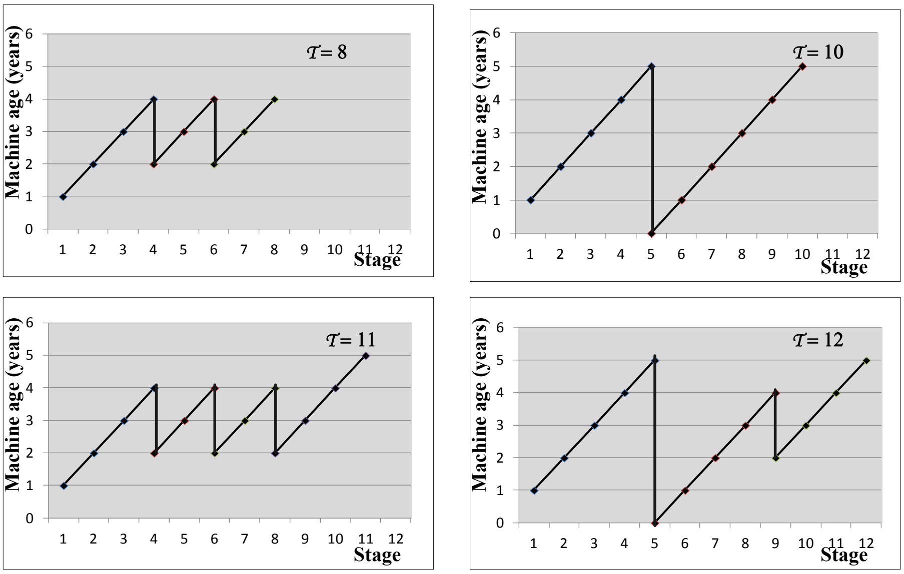

Figure 7 Machine's age at stage j due to decisions x_j for different lengths of $\mathcal{T}$.

5 Conclusions

In this paper, we discuss a simple optimal overhaul-replacement model for a repairable machine sold under warranty, from the buyer's point of view. The machine is a means of production that should be operated for a finite horizon planning. The buyer wants to minimize the total ownership costs over the planning horizon by considering overhaul in order to extend machine life. At the same time, the buyer also wants the benefit from the warranty that is attached to the decision to replace the machine. We develop a dynamic programming formulation to determine overhaul-replacement policies that minimize the total ownership costs over the planning horizon considered. The length of the warranty and the rejuvenation effect of overhaul were modeled as integer multiplications of the periodic evaluation span. We use a numerical approach to obtain the solution and solved the model with a computer program written in Visual Basic.

The model provides valuable insights into the structure of the optimal solution for machine replacement under warranty in a more complex situation. Using this model, the buyer can find the periodic optimal policy and the intended minimum cost as well as the replacement schedule and periodic amount of capital required. By understanding the interaction effects of the advantages of the warranty and the benefits of overhaul, the buyer can also attempt to make an overhaul improvement and avoid costly frequent machine replacement. Manufacturers can also use this model, in particular to understand the customer's underlying decision to replace in the context of the machine's life cycle and the customer's purchase behavior. The manufacturer could consider improving the offered benefit, such as extending the warranty length, to promote sales. The replacement overhaul-model for a warranted machine with a Markovian deterioration model and overhaul rejuvenation being a function of the machine's age are other topics of research currently being investigated by the authors.

Nomenclature

The following notation was used in developing the proposed model.

c_1	: minimal repair cost charged per failure during the warranty
c_2	: minimal repair cost charged per failure after the warranty expires
c_3	: overhaul cost
c_4	: replacement cost
$e(t)$	: salvage (or trade-in) value at age t
$c_4(t)$	: replacement cost at age t
$h(t, t+s)$	: expected number of failures during s for a machine with age t
j	: evaluation point at the beginning of any operation interval, j= 0,1,…N
N	: evaluation number during respective planning period (N integer, $s = \mathcal{T}/N$)
s	: operation interval (between evaluation points)
t	: machine age at j
$\mathcal{T}$	: planning horizon
w	: the warranty period, $w = n \cdot s$ (n=1, 2,…)
x_j	: decision alternatives at j, x_j = {K, O, R}
δ	: machine age reduction due to overhaul decision
$\lambda(\tau)$	: failure intensity function

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